Loop-abort faults on supersingular isogeny cryptosystems

Alexandre Gélin Benjamin Wesolowski

Laboratoire d'Informatique de Paris 6 – Sorbonne Universités UPMC, France

École Polytechnique Fédérale de Lausanne, EPFL IC LACAL, Switzerland

PQCrypto 2017 – Utrecht

2017/06/26

Supersingular-Isogeny Public-key Cryptography

- Introduced by Jao, De Feo, and Plût in 2011

Supersingular-Isogeny Public-key Cryptography

- Introduced by Jao, De Feo, and Plût in 2011
- Based on the same problem as the hash function of [CLG06]

Supersingular-Isogeny Public-key Cryptography

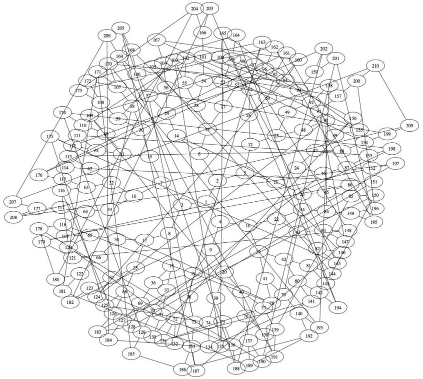
- Introduced by Jao, De Feo, and Plût in 2011
- Based on the same problem as the hash function of [CLG06]

The isogeny graph of a supersingular elliptic curve:

Supersingular-Isogeny Public-key Cryptography

- Introduced by Jao, De Feo, and Plût in 2011
- Based on the same problem as the hash function of [CLG06]

The isogeny graph of a supersingular elliptic curve:



Supersingular elliptic curves

Definition

A supersingular elliptic curve is a curve E defined over $\mathbf{F}_{p^k}$ such that

$$\#E\left(\mathbf{F}_{p^k}\right) = 1 \bmod p.$$

Supersingular elliptic curves

Definition

A supersingular elliptic curve is a curve E defined over $\mathbf{F}_{p^k}$ such that

$$\#E\left(\mathbf{F}_{p^k}\right) = 1 \bmod p.$$

Interesting properties:

- All supersingular elliptic curves can be defined over $\mathbf{F}_{p^2}$
- About $\frac{p}{12}$ supersingular elliptic curves, up to isomorphism

Isogenies

Definition

An isogeny ϕ between two elliptic curves E_1 and E_2 is a surjective group homomorphism with a finite kernel. The degree is defined by

$$\deg \phi = \#\mathrm{Ker}\ \phi.$$

Isogenies

Definition

An isogeny ϕ between two elliptic curves E_1 and E_2 is a surjective group homomorphism with a finite kernel. The degree is defined by

$$\deg \phi = \#\mathrm{Ker}\ \phi.$$

Interesting properties:

- $G \subset E_1 \implies$ a unique E_2 and ϕ such that

$$\phi : E_1 \to E_2 \quad \text{and} \quad \mathrm{Ker}\ \phi = G$$

- $E_2 = E/G$ is obtained in $O(\deg \phi)$

Key-Exchange Protocol

- A prime p such that $p+1=\ell_A^n\ell_B^m$

Key-Exchange Protocol

- A prime p such that $p+1 = \ell_A^n \ell_B^m$
- A supersingular elliptic curve E with $\ell_A^n \ell_B^m$ points

E

Key-Exchange Protocol

- A prime p such that $p+1=\ell_A^n\ell_B^m$
- A supersingular elliptic curve E with $\ell_A^n\ell_B^m$ points
- A point R_A chosen randomly in $E[\ell_A^n]$

E

Key-Exchange Protocol

- A prime p such that $p+1 = \ell_A^n \ell_B^m$
- A supersingular elliptic curve E with $\ell_A^n \ell_B^m$ points
- A point R_A chosen randomly in $E[\ell_A^n]$

  $\longrightarrow (m_A, n_A) \in \{1, \ldots, \ell_A^n\}^2$ random,
  $R_A = m_A P_A + n_A Q_A$ for $\langle P_A, Q_A \rangle = E[\ell_A^n]$

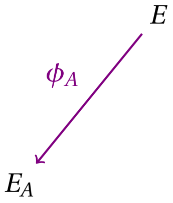
E

Key-Exchange Protocol

- A prime p such that $p+1 = \ell_A^n \ell_B^m$
- A supersingular elliptic curve E with $\ell_A^n \ell_B^m$ points
- A point R_A chosen randomly in $E[\ell_A^n]$

$\longrightarrow (m_A, n_A) \in \{1, \ldots, \ell_A^n\}^2$ random,
$R_A = m_A P_A + n_A Q_A$ for $\langle P_A, Q_A \rangle = E[\ell_A^n]$

$\Longrightarrow$ the curve $E_A = E/\langle R_A \rangle$ and $\phi_A : E \to E_A$


$$\begin{tikzcd} & E \arrow[dl, "\phi_A"'] \\ E_A & \end{tikzcd}$$

Key-Exchange Protocol

- A prime p such that $p+1 = \ell_A^n \ell_B^m$
- A supersingular elliptic curve E with $\ell_A^n \ell_B^m$ points
- A point R_A chosen randomly in $E[\ell_A^n]$

  $\longrightarrow (m_A, n_A) \in \{1, \dots, \ell_A^n\}^2$ random,
  $R_A = m_A P_A + n_A Q_A$ for $\langle P_A, Q_A \rangle = E[\ell_A^n]$

  $\Longrightarrow$ the curve $E_A = E/\langle R_A \rangle$ and $\phi_A : E \to E_A$
- A point $R_B = m_B P_B + n_B Q_B$ random in $E[\ell_B^m] = \langle P_B, Q_B \rangle$,
  the curve $E_B = E/\langle R_B \rangle$ and $\phi_B : E \to E_B$

$$\begin{tikzcd}
& E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A & & E_B
\end{tikzcd}$$

Key-Exchange Protocol

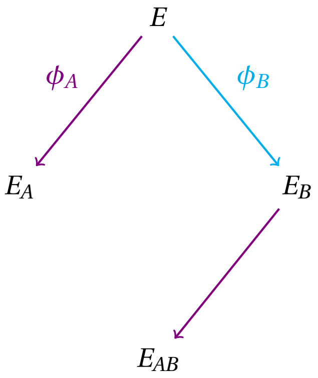
- Bob sends $\big(E_B, \phi_B(P_A), \phi_B(Q_A)\big)$ where $\langle \phi_B(P_A), \phi_B(Q_A) \rangle = E_B[\ell_A^n]$

$$\begin{tikzcd} & E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\ E_A & & E_B \end{tikzcd}$$

Key-Exchange Protocol

- Bob sends $\big(E_B, \phi_B(P_A), \phi_B(Q_A)\big)$ where $\langle \phi_B(P_A), \phi_B(Q_A)\rangle = E_B[\ell_A^n]$

- Alice computes $E_{AB} = E_B / \langle m_A \phi_B(P_A) + n_A \phi_B(Q_A)\rangle$


$$\begin{tikzcd} & E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\ E_A & & E_B \arrow[dl] \\ & E_{AB} & \end{tikzcd}$$

Key-Exchange Protocol

- Bob sends $\big(E_B, \phi_B(P_A), \phi_B(Q_A)\big)$
  where $\langle \phi_B(P_A), \phi_B(Q_A) \rangle = E_B[\ell_A^n]$
- Alice computes $E_{AB} = E_B / \langle m_A \phi_B(P_A) + n_A \phi_B(Q_A) \rangle$
- Bob computes $E_{BA} = E_A / \langle m_B \phi_A(P_B) + n_B \phi_A(Q_B) \rangle$

$$\begin{tikzcd}
 & E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
 & E_{AB} &
\end{tikzcd}$$

Key-Exchange Protocol

- Bob sends $\big(E_B, \phi_B(P_A), \phi_B(Q_A)\big)$
  where $\langle \phi_B(P_A), \phi_B(Q_A)\rangle = E_B[\ell_A^n]$
- Alice computes $E_{AB} = E_B / \langle m_A \phi_B(P_A) + n_A \phi_B(Q_A)\rangle$
- Bob computes $E_{BA} = E_A / \langle m_B \phi_A(P_B) + n_B \phi_A(Q_B)\rangle$
- $E_{AB} \simeq E/\langle R_A, R_B\rangle \simeq E_{BA}$ so $j(E_{AB}) = j(E_{BA})$

$$\begin{tikzcd}
& E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
& E_{AB} &
\end{tikzcd}$$

Key-Exchange Protocol

- Bob sends $\big(E_B, \phi_B(P_A), \phi_B(Q_A)\big)$
  where $\langle \phi_B(P_A), \phi_B(Q_A)\rangle = E_B[\ell_A^n]$
- Alice computes $E_{AB} = E_B/\langle m_A\phi_B(P_A) + n_A\phi_B(Q_A)\rangle$
- Bob computes $E_{BA} = E_A/\langle m_B\phi_A(P_B) + n_B\phi_A(Q_B)\rangle$
- $E_{AB} \simeq E/\langle R_A, R_B\rangle \simeq E_{BA}$ so $j(E_{AB}) = j(E_{BA})$

$\Longrightarrow$ $j(E_{AB})$ secret shared by Alice and Bob ☺

$$\begin{tikzcd}
 & E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
 & E_{AB} &
\end{tikzcd}$$

Position of the problem

Path-finding problem

Given two isogenous curves E_1 and E_2, find an isogeny between them of degree ℓ_A^n.

Position of the problem

Path-finding problem

Given two isogenous curves E_1 and E_2, find an isogeny between them of degree ℓ_A^n.

- Equivalent to find a path of fixed length in the isogeny graph

Position of the problem

Path-finding problem

Given two isogenous curves E_1 and E_2, find an isogeny between them of degree ℓ_A^n.

- Equivalent to find a path of fixed length in the isogeny graph
- Brute-force attack in $O(\ell_A^n) \approx O(\sqrt{p})$

Position of the problem

Path-finding problem

Given two isogenous curves E_1 and E_2, find an isogeny between them of degree ℓ_A^n.

- Equivalent to find a path of fixed length in the isogeny graph
- Brute-force attack in $O\left(\ell_A^n\right) \approx O\left(\sqrt{p}\right)$
- Claw finding: Find a collision in $O\left(\ell_A^{\frac{n}{2}}\right) \approx O\left(\sqrt[4]{p}\right)$

Attack framework

- Alice uses a static private key (m_A, n_A)

$$\begin{tikzcd} & E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\ E_A \arrow[dr] & & E_B \arrow[dl] \\ & E_{AB} & \end{tikzcd}$$

Attack framework

- Alice uses a static private key (m_A, n_A)

  $\implies E_A$ and ϕ_A can be precomputed

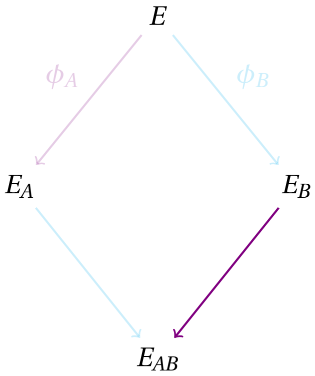
$$\begin{tikzcd}
 & E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
 & E_{AB} &
\end{tikzcd}$$

Attack framework

- Alice uses a static private key (m_A, n_A)

  $\Longrightarrow E_A$ and ϕ_A can be precomputed

- The attacker plays the role of Bob



$$\begin{tikzcd}
& E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
& E_{AB} &
\end{tikzcd}$$

Attack framework

- Alice uses a static private key (m_A, n_A)

  $\Longrightarrow E_A$ and ϕ_A can be precomputed

- The attacker plays the role of Bob

- Focus on the isogeny from E_B to $E_B / \langle m_A P'_A + n_A Q'_A \rangle$,

  where $P'_A = \phi_B(P_A)$ and $Q'_A = \phi_B(Q_A)$

$$\begin{tikzcd}
 & E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
 & E_{AB} &
\end{tikzcd}$$

Attack framework

- Alice uses a static private key (m_A, n_A)

  $\Longrightarrow E_A$ and ϕ_A can be precomputed

- The attacker plays the role of Bob

- Focus on the isogeny from E_B to $E_B/\langle m_A P'_A + n_A Q'_A \rangle$, where $P'_A = \phi_B(P_A)$ and $Q'_A = \phi_B(Q_A)$

- Previous active attack [GPST16]:
  - Idea: Provide dishonest points $(\widetilde{P}_A, \widetilde{Q}_A)$ instead of (P'_A, Q'_A)

$$\begin{tikzcd}
& E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
& E_{AB} &
\end{tikzcd}$$

Attack framework

- Alice uses a static private key (m_A, n_A)

  $\Longrightarrow E_A$ and ϕ_A can be precomputed

- The attacker plays the role of Bob

- Focus on the isogeny from E_B to $E_B/\langle m_A P'_A + n_A Q'_A \rangle$,

  where $P'_A = \phi_B(P_A)$ and $Q'_A = \phi_B(Q_A)$

- Previous active attack [GPST16]:
  - Idea: Provide dishonest points $(\widetilde{P}_A, \widetilde{Q}_A)$ instead of (P'_A, Q'_A)
  - Countermeasure: Validation method verifies the correctness of the inputs (Fujisaki-Okamoto transform)

$$\begin{tikzcd}
& E \arrow[dl, "\phi_A"'] \arrow[dr, "\phi_B"] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
& E_{AB} &
\end{tikzcd}$$

How is computed the isogeny ?

- Degree ℓ_A^n: Vélu's formulae $\Rightarrow O(\ell_A^n)$ ☹

How is computed the isogeny ?

- Degree ℓ_A^n: Vélu's formulae $\Rightarrow O(\ell_A^n)$ ☹
- Decompose and iterate $\Rightarrow n \cdot O(\ell_A)$ ☺

$$E_B = E_0 \to E_1 \to \cdots \to E_{n-1} \to E_n = E_{AB}$$

where each $\to$ is a degree-ℓ_A isogeny

How is computed the isogeny ?

- Degree ℓ_A^n: Vélu's formulae $\Rightarrow O(\ell_A^n)$ ☹

- Decompose and iterate $\Rightarrow n \cdot O(\ell_A)$ ☺

$$E_B \quad = \quad E_0 \quad \to \quad E_1 \quad \to \quad \cdots \quad \to \quad E_{n-1} \quad \to \quad E_n \quad = \quad E_{AB}$$

  where each $\to$ is a degree-ℓ_A isogeny

- $R_0 = m_A P'_A + n_A Q'_A$ and for $1 \leq k \leq n-1$,

$$E_{k+1} = E_k / \langle \ell_A^{n-k-1} R_k \rangle \qquad \phi_{k+1} : E_k \to E_{k+1} \qquad R_{k+1} = \phi_{k+1}(R_k)$$

How is computed the isogeny ?

- Degree ℓ_A^n: Vélu's formulae $\Rightarrow O(\ell_A^n)$ ☹

- Decompose and iterate $\Rightarrow n \cdot O(\ell_A)$ ☺

$$E_B = E_0 \to E_1 \to \cdots \to E_{n-1} \to E_n = E_{AB}$$

where each $\to$ is a degree-ℓ_A isogeny

- $R_0 = m_A P'_A + n_A Q'_A$ and for $1 \leq k \leq n-1$,

$$E_{k+1} = E_k / \langle \ell_A^{n-k-1} R_k \rangle \qquad \phi_{k+1} : E_k \to E_{k+1} \qquad R_{k+1} = \phi_{k+1}(R_k)$$

- $E_n = E_{AB} = E_B / \langle R_0 \rangle$ and $\phi = \phi_n \circ \cdots \circ \phi_1$

Loop-abort fault attacks

- Introduced for pairing-based cryptography

  Used recently in the context of lattice-based signature schemes

Loop-abort fault attacks

- Introduced for pairing-based cryptography

  Used recently in the context of lattice-based signature schemes

- Inject a fault that induces an early-abort in the loop

Loop-abort fault attacks

- Introduced for pairing-based cryptography

  Used recently in the context of lattice-based signature schemes

- Inject a fault that induces an early-abort in the loop

- Proven feasible in practice [Blömer *et al.*]

Loop-abort fault attacks

- Introduced for pairing-based cryptography

  Used recently in the context of lattice-based signature schemes

- Inject a fault that induces an early-abort in the loop

- Proven feasible in practice [Blömer *et al.*]

- Implementations of SIDH on embedded devices already exist

The attack $\ell_A = 2$

- Need an oracle to compare Alice's outputs with what the attacker computes

The attack $\ell_A = 2$

- Need an oracle to compare Alice's outputs with what the attacker computes
- After k iterations, Alice has computed the intermediate curve

$$E_k = E_B / \langle 2^{n-k}(m_A P'_A + n_A Q'_A) \rangle$$

The attack $\ell_A = 2$

- Need an oracle to compare Alice's outputs with what the attacker computes
- After k iterations, Alice has computed the intermediate curve

$$E_k = E_B / \langle 2^{n-k}(m_A P'_A + n_A Q'_A) \rangle$$

- Guess strategy: first step, $k = 1$

The attack $\ell_A = 2$

- Need an oracle to compare Alice's outputs with what the attacker computes
- After k iterations, Alice has computed the intermediate curve

$$E_k = E_B / \langle 2^{n-k}(m_A P'_A + n_A Q'_A) \rangle$$

- Guess strategy: first step, $k = 1$
  - if m_a is even, then Ker $\phi_1 = \langle 2^{n-1} Q'_A \rangle$

The attack $\ell_A = 2$

- Need an oracle to compare Alice's outputs with what the attacker computes
- After k iterations, Alice has computed the intermediate curve

$$E_k = E_B / \langle 2^{n-k}(m_A P'_A + n_A Q'_A) \rangle$$

- Guess strategy: first step, $k = 1$
  - if m_a is even, then Ker $\phi_1 = \langle 2^{n-1} Q'_A \rangle$
  - if n_a is even, then Ker $\phi_1 = \langle 2^{n-1} P'_A \rangle$

The attack

- Need an oracle to compare Alice's outputs with what the attacker computes
- After k iterations, Alice has computed the intermediate curve

$$E_k = E_B / \langle 2^{n-k}(m_A P'_A + n_A Q'_A)\rangle$$

- Guess strategy: first step, $k = 1$
  - if m_a is even, then Ker $\phi_1 = \langle 2^{n-1} Q'_A \rangle$
  - if n_a is even, then Ker $\phi_1 = \langle 2^{n-1} P'_A \rangle$
  - if both are odd, then Ker $\phi_1 = \langle 2^{n-1}(P'_A + Q'_A)\rangle$

The attack

- Need an oracle to compare Alice's outputs with what the attacker computes
- After k iterations, Alice has computed the intermediate curve

$$E_k = E_B / \langle 2^{n-k}(m_A P'_A + n_A Q'_A) \rangle$$

- Guess strategy: first step, $k = 1$
  - if m_a is even, then Ker $\phi_1 = \langle 2^{n-1} Q'_A \rangle$

    $\Longrightarrow (m_A, n_A)$ equivalent to $(a, 1)$ for $a = \frac{m_A}{n_A}$ and a even
  - if n_a is even, then Ker $\phi_1 = \langle 2^{n-1} P'_A \rangle$
  - if both are odd, then Ker $\phi_1 = \langle 2^{n-1}(P'_A + Q'_A) \rangle$

The attack

- Need an oracle to compare Alice's outputs with what the attacker computes
- After k iterations, Alice has computed the intermediate curve

$$E_k = E_B / \langle 2^{n-k}(m_A P'_A + n_A Q'_A) \rangle$$

- Guess strategy: first step, $k = 1$
    - if m_a is even, then Ker $\phi_1 = \langle 2^{n-1} Q'_A \rangle$

      $\implies (m_A, n_A)$ equivalent to $(a, 1)$ for $a = \frac{m_A}{n_A}$ and a even
    - if n_a is even, then Ker $\phi_1 = \langle 2^{n-1} P'_A \rangle$

      $\implies (m_A, n_A)$ equivalent to $(1, a)$ for $a = \frac{n_A}{m_A}$ and a even
    - if both are odd, then Ker $\phi_1 = \langle 2^{n-1}(P'_A + Q'_A) \rangle$

The attack $\ell_A = 2$

- Need an oracle to compare Alice's outputs with what the attacker computes
- After k iterations, Alice has computed the intermediate curve

$$E_k = E_B / \langle 2^{n-k}(m_A P'_A + n_A Q'_A) \rangle$$

- Guess strategy: first step, $k = 1$
  - if m_a is even, then Ker $\phi_1 = \langle 2^{n-1} Q'_A \rangle$

    $\implies (m_A, n_A)$ equivalent to $(a, 1)$ for $a = \frac{m_A}{n_A}$ and a even
  - if n_a is even, then Ker $\phi_1 = \langle 2^{n-1} P'_A \rangle$

    $\implies (m_A, n_A)$ equivalent to $(1, a)$ for $a = \frac{n_A}{m_A}$ and a even
  - if both are odd, then Ker $\phi_1 = \langle 2^{n-1}(P'_A + Q'_A) \rangle$

    $\implies (m_A, n_A)$ equivalent to $(1, a)$ for $a = \frac{n_A}{m_A}$ and a odd

The attack $\ell_A = 2$

- Subsequent steps: we assume the key of the form $(1, a)$

The attack $\ell_A = 2$

- Subsequent steps: we assume the key of the form $(1, a)$
- We know the $k-1$ least significant bits

The attack $\ell_A = 2$

- Subsequent steps: we assume the key of the form $(1, a)$
- We know the $k-1$ least significant bits
- The k-th bit is either 0 or 1, *i.e.*,

$$E_k = E_B / \left\langle 2^{n-k}\left(P'_A + (a \bmod 2^{k-1})Q'_A\right)\right\rangle \quad \text{or} \quad E_k = E_B / \left\langle 2^{n-k}\left(P'_A + (a \bmod 2^{k-1} + 2^{k-1})Q'_A\right)\right\rangle$$

The attack

- Subsequent steps: we assume the key of the form $(1, a)$
- We know the $k-1$ least significant bits
- The k-th bit is either 0 or 1, *i.e.*,

$$E_k = E_B / \left\langle 2^{n-k}\left(P'_A + (a \bmod 2^{k-1})Q'_A\right)\right\rangle \quad \text{or} \quad E_k = E_B / \left\langle 2^{n-k}\left(P'_A + (a \bmod 2^{k-1} + 2^{k-1})Q'_A\right)\right\rangle$$

- Make a guess and recover the k-th bit of a

The attack

- Subsequent steps: we assume the key of the form $(1, a)$
- We know the $k-1$ least significant bits
- The k-th bit is either 0 or 1, *i.e.*,

$$E_k = E_B / \left\langle 2^{n-k}\left(P'_A + (a \bmod 2^{k-1})Q'_A\right)\right\rangle \quad \text{or} \quad E_k = E_B / \left\langle 2^{n-k}\left(P'_A + (a \bmod 2^{k-1} + 2^{k-1})Q'_A\right)\right\rangle$$

- Make a guess and recover the k-th bit of a
- Conclusion: full-key recovery by iterating this process

Analysis

- n bits recovered in n interactions with the victim $\Longrightarrow$ n faults injected

Analysis

- n bits recovered in n interactions with the victim $\Longrightarrow$ n faults injected
- if the success probability μ of the fault injection is not 1,

Analysis

- n bits recovered in n interactions with the victim $\Longrightarrow$ n faults injected
- if the success probability μ of the fault injection is not 1,
  - about $\frac{n}{\mu}$ faults injected if the success can be detected

Analysis

- n bits recovered in n interactions with the victim $\Longrightarrow$ n faults injected
- if the success probability μ of the fault injection is not 1,
  - about $\frac{n}{\mu}$ faults injected if the success can be detected
  - about $\frac{2n}{\mu}$ otherwise

Analysis

- n bits recovered in n interactions with the victim $\Longrightarrow$ n faults injected
- if the success probability μ of the fault injection is not 1,
  - about $\frac{n}{\mu}$ faults injected if the success can be detected
  - about $\frac{2n}{\mu}$ otherwise
- Alternative with less faults assuming a stronger oracle

Thanks

Bedankt